Solutions to JEE Advanced Home Practice Test -8 |JEE 2024 | Paper-1

PHYSICS

1.(C)

2.(C) $\dfrac{Q}{4\pi \in_0 r_0^2} = \dfrac{\sigma}{2 \in_0}$

$Q = 2\pi\sigma r_0^2 = 1\mu C$

$\dfrac{\sigma}{2 \in_0} = \dfrac{\lambda}{2\pi \in_0 r_0}$

$\lambda = \pi\sigma = 0.5\mu C / m$

$\dfrac{Q}{4\pi \in_0 \times \left(\dfrac{1}{2}\right)^2} = k\dfrac{\lambda}{2\pi \in_0 \left(\dfrac{1}{2}\right)}$

$Q = 1 = k \times 0.5 \Rightarrow k = 2$

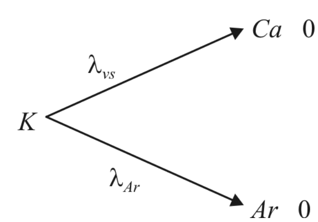
3.(A) $dQ = HdT$; $\dfrac{dQ}{dt} = P = H\dfrac{dT}{dt} = HT_0 2\beta t$

$H.2\beta T_0 t = P$

$\because \quad T = T_0\left(1+\beta t^2\right) \quad \Rightarrow \quad 1+\beta t^2 = \dfrac{T}{T_0}K \quad \Rightarrow \quad t^2 = \left(\dfrac{T}{T_0}-1\right)\dfrac{1}{\beta} \quad \Rightarrow \quad t = \sqrt{\dfrac{T-T_0}{\beta T_0}}$

So, $H = \dfrac{P}{2\beta T_0\sqrt{\dfrac{T-T_0}{\beta T_0}}} = \dfrac{P/2}{\sqrt{\beta T_0\left(T-T_0\right)}}$

4.(B) $t = 0$



$\dfrac{dN}{dt} = -\left(\lambda_1+\lambda_2\right)\times N$

$\log_e\left(\dfrac{N}{N_0}\right) = -\left(\lambda_1+\lambda_2\right)t$

$\because \quad \dfrac{N_0-N}{N} = 9 \quad \Rightarrow \quad N_0 - N = 9N \quad \Rightarrow \quad N = \dfrac{N_0}{10}$

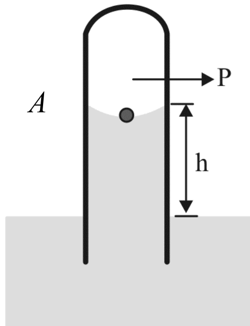
$2.3\log_{10}\left(\dfrac{N_0/10}{N_0}\right) = -5\times10^{-10}t$

$2.3\times-1 = -5\times10^{-10}t$

$t = \dfrac{2.3}{5}\times10^{10}\text{ years} = 0.46\times10^{10}\,years = 4.6\times10^{9}\,years$

5.(BC) When the tube is brought into contact with ,it is filled with air at atmospheric pressure. When water rises to a height h, the air pressure (P) is given by

$\therefore \quad P = \dfrac{P_o L}{L-h}$



Radius of curvature of meniscus $R = r,$ since contact angle is zero.

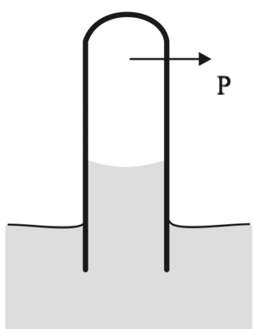
Pressure at A is $P_A = P_o - \rho g h$

$\therefore \quad P = P_A + \dfrac{2T}{r} \qquad \therefore \quad \dfrac{P_o L}{L-h} = P_o - \rho g h + \dfrac{2T}{r}$

$\Rightarrow \quad \dfrac{P_0 h}{L-h} = \dfrac{2T}{r} - \rho g h \quad \Rightarrow \quad L - h = \dfrac{P_0 h}{\dfrac{2T}{r} - \rho g h} \quad \Rightarrow \quad L = h + \dfrac{P_0 r h}{2T - \rho g h r}$

Surface tension force on outer wall of the tube is $2\pi R T(\downarrow)$

Surface tension force (at meniscus) on the inner tube wall is $2\pi r T(\downarrow)$



Force due to air pressure inside the tube $\pi r^2 P(\uparrow)$

Force due to air pressure inside the tube $\pi R^2 P_0(\downarrow)$

For equilibrium of the tube, let the upward force needed be F

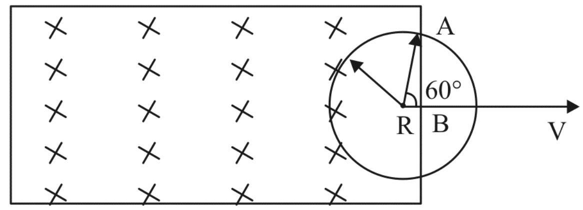
$F + P\pi r^2 = Mg + \pi R^2 P_0 + 2\pi R T + 2\pi r T$

$F = Mg + \pi R^2 P_0 - \dfrac{P_0 L \pi r^2}{L-h} + 2\pi(R+r)T$

$F = Mg + \pi P_0\left[R^2 - \dfrac{L r^2}{L-h}\right] + 2\pi(R+r)T$

6.(ABC)

At any time t



$$I=\frac{\varepsilon}{\lambda 2\pi R}=\frac{B(2AB)v}{\lambda 2\pi R}$$

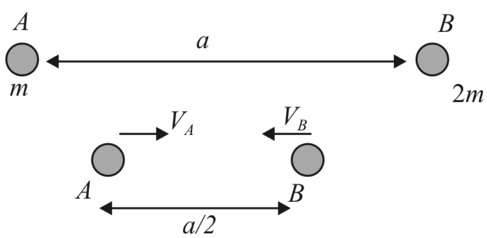
$$I=\frac{Bv}{\lambda\pi R}\sqrt{R^2-\left(\frac{R}{2}-vt\right)^2}$$

$$I=\frac{Bv}{\lambda\pi}\sqrt{1-\left(\frac{1}{2}-\frac{vt}{R}\right)^2}$$

$$I=\frac{2Bv}{2\lambda\pi R}\sqrt{R^2-\frac{R^2}{4}+Rvt-v^2t^2}$$

$$I=\frac{Bv}{2\lambda\pi R}\sqrt{3R^2+4Rvt-4v^2t^2}$$

7.(AD)



There is no external force acting on system.

$\vec{F}_{ext}=0$

by conservation of momentum.

$0+0=mv_A+(2m)(-v_B)$

$mv_A=2mv_B$

$v_A=2v_B$

Conservation of energy.

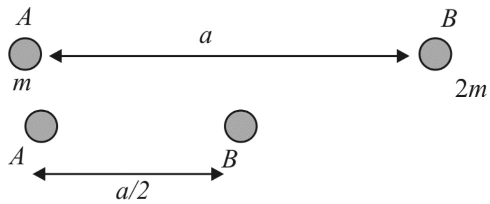
$$T.E_i=T.E_f \qquad -\frac{Gm2m}{a}+0=-\frac{Gm2m}{a/2}+\frac{1}{2}mv_A^2+\frac{1}{2}(2m)v_B^2$$

$$-\frac{2Gm^2}{a}+\frac{4Gm^2}{a}=\frac{1}{2}mv_A^2+\frac{1}{2}(2m)v_B^2; \qquad \frac{2Gm^2}{a}=\frac{1}{2}m(2v_B)^2+\frac{1}{2}(2m)v_B^2$$

$$\frac{2Gm^2}{a}=\left(mv_B^2\right)\left[\frac{4}{2}+\frac{2}{2}\right]\Rightarrow v_B^2=\frac{2Gm}{3a}; \qquad v_B=\sqrt{\frac{2Gm}{3a}} \quad\Rightarrow\quad v_A=2v_B=\sqrt{\frac{8Gm}{3a}}$$

$$\text{Acceleration of A }=\frac{F}{m}=\frac{\frac{Gm2m}{(a/2)^2}}{m} \quad\Rightarrow\quad a_A=\frac{8Gm}{a^2}$$

Combined K.E. $=\frac{1}{2}mv_A^2+\frac{1}{2}(2m)v_B^2=\frac{1}{2}m\frac{8Gm}{3a}+\frac{1}{2}(2m)\frac{2Gm}{3a}=\frac{Gm^2}{3a}\left[\frac{8}{2}+\frac{4}{2}\right]=\frac{Gm^2 6}{3a}=\frac{2Gm^2}{a}$



Distance of c.o.m from A $=\frac{2ma}{3m}=\frac{2a}{3}$

Distance of c.o.m form A $=\frac{2m(a/2)}{3m}=\frac{a}{3}$

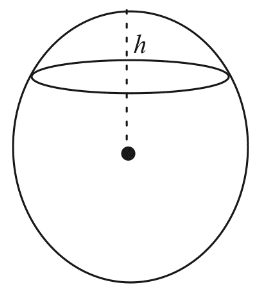
Distance moved by 'A' $=\frac{2a}{3}-\frac{a}{3}=\frac{a}{3}$.

8.(ABD)

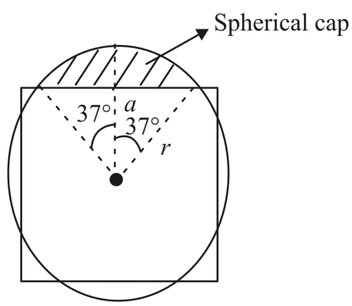
$r=$ radius of sphere

$h=$ height of spherical cap

Volume of spherical cap $=\frac{\pi h^2}{3}(3r-h)$



(A)



$\because \quad r\cos 37°=a\,; \qquad r\cdot\frac{4}{5}=a \quad\Rightarrow\quad r=\frac{5a}{4}$

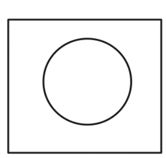
$H=$ Height of spherical cap $=r-a=\frac{5a}{4}-a=\frac{a}{4}$

Volume of spherical cap $=\frac{\pi h^3}{3}(3r-h)=\frac{\pi}{3}\frac{a^2}{16}\left(3\times\frac{5a}{4}-\frac{a}{4}\right)=\frac{\pi a^2}{48}\times\frac{14a}{4}=\frac{7}{96}\pi a^3$

Volume enclosed in Gaussian surface $=$ Volume of sphere $-6\times$ volume of spherical cap

$=\frac{4}{3}\pi r^3-6\times\frac{7}{96}\pi a^3=\frac{4}{3}\pi\left(\frac{5a}{4}\right)^3-\frac{42}{96}\pi a^3=\frac{13}{6}\pi a^3\,; \qquad \text{Flux }=\left(\frac{\rho 13}{\epsilon_0\, 6}\pi a^3\right)$

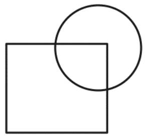
(B)



$r=\frac{a}{2}$

(Volume enclosed in Gaussian surface) $=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\frac{a^3}{8}=\frac{\pi a^3}{6}\,; \qquad \text{Flux }=\frac{\rho\pi a^3}{6\,\epsilon_0}$

(D)

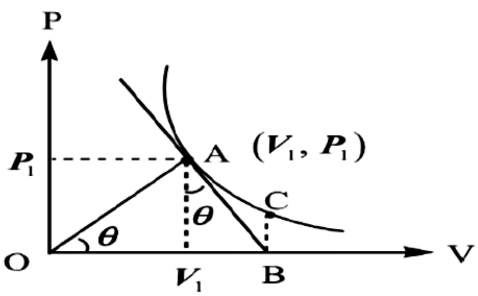


$\because \quad r = a$

Volume enclosed in Gaussian surface $= \frac{1}{8} \times \frac{4}{3}\pi r^3 = \frac{\pi}{6} a^3$

Flux $= \frac{\rho \pi a^3}{6 \in_0}$

9.(AC)



$\tan\theta = \frac{P_1}{V_1}$

Slope of curve at A is $\tan(90+\theta) = -\frac{P_1}{V_1}$ or

$\cot\theta = \frac{P_1}{V_1}$

$\tan\theta = \cot\theta \Rightarrow \theta = 45 \Rightarrow V_B = 2V_1 \Rightarrow P_C = \frac{P_1}{2}$

Area $= \frac{1}{2}(OB)P_1 = \frac{1}{2} 2V_1 P_1 = P_1 V_1 = nRT$

10.(AC)

$\frac{\mu}{v} - \frac{1}{\infty} = \frac{(\mu - 1)}{10}$

$v = \frac{10\mu}{\mu - 1} = \frac{10}{1 - \frac{1}{\mu}}$

$v_v < v_r$

$v_v = \frac{10}{0.61} \times 1.61 \qquad \Rightarrow \qquad v_r = \frac{10}{0.6} \times 1.6$

$v_v - v_r = \frac{16.1}{0.61} - \frac{16}{0.6} = \frac{16.1 \times 0.6 - 16 \times 0.161}{0.6 \times 0.61} = 0.27\ cm$

11.(B)

Given

$L = x^{\alpha}$ …… (i)

$LT^{-1} = x^{\beta}$ …… (ii)

$LT^{-2} = x^{P}$ …… (iii)

$MLT^{-1} = x^{q}$ …… (iv)

$MLT^{-2} = x^{r}$ …… (v)

$\frac{(i)}{(ii)} \Rightarrow T = x^{\alpha-\beta}$

From (iii)

$\frac{x^{\alpha}}{x^{2(\alpha-\beta)}} = x^{p}$

$\Rightarrow \alpha + p = 2\beta$ …… (A)

From (iv)

$M = x^{q-\beta}$

From (v)

$\Rightarrow x^{q} = x^{r}x^{\alpha-\beta}$

$\Rightarrow \alpha + r - q = \beta$ ……(vi)

Replacing value $'\alpha'$ in equation (vi) from (A)

$2\beta - p + r - q = \beta$

$\Rightarrow p + q - r = \beta$ …… (B)

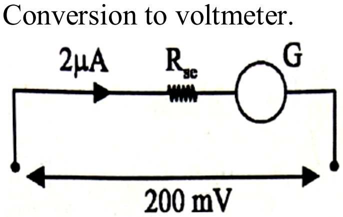
Replacing value of β in equation (vi) from (A)

$2\alpha + 2r - 2q = \alpha + p$

$\alpha = p + 2q - 2r$

12.(ABD)

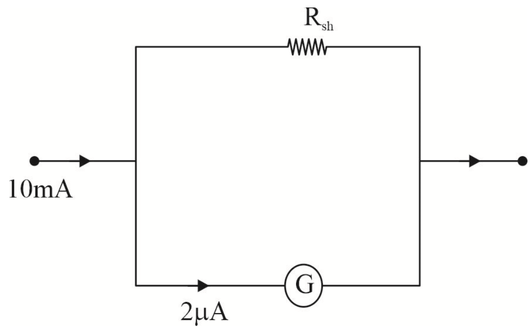
Conversion to voltmeter.



$200\times10^{-3} = 2\times10^{-6}(R_{se}+10) \Rightarrow R_{se}+10 = 100\times10^{3} = 100k\Omega$

Resistance of voltmeter $= R_v = R_{se} + 10 = 100k\Omega$

Conversion to ammeter:

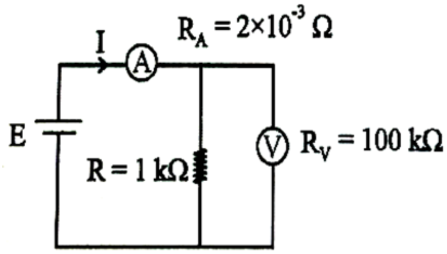


$$\Rightarrow \quad 10\times 2\times 10^{-6} = \left(10\times 10^{-3} - 2\times 10^{-6}\right)R_{sh}$$

$$\Rightarrow \quad 20 = \left(10\times 10^{3} - 2\right)R_{sh} \quad \Rightarrow \quad R_{sh} \cong \frac{20}{10\times 10^{3}} = 2\times 10^{-3}\Omega$$

Resistance of ammeter $= 2\times 10^{-3}\Omega$

Circuit:



$$\because \quad V = I.\frac{R.R_v}{R+R_v}$$

$$R_{measured} = \frac{V}{I} = \frac{RR_v}{R+R_v} = \frac{1\times 100}{101}k\Omega \cong 990.1\Omega$$

Note measured value is independent of nature of cell.

13.(5) $W_F = \mu mg\times R + mgh = (\mu+1)mgR = \left(1+\frac{1}{4}\right)\times 2\times 10\times 0.2 = \frac{5}{4}\times 2\times 2 = 5J$

14.(0.45)

Total increase in length of rods $= L\alpha\Delta T + \frac{L}{2}\alpha\Delta T = 3\frac{L}{2}\alpha\Delta T$

Let the compression in spring A is x_A, B is x_B and C is

$x_C \quad \Rightarrow \quad k_A x_A = k_B x_B = k_C x_C$

$\Rightarrow \quad kx_A = 2kx_B = kx_C \quad \Rightarrow \quad x_A = 2x_B = x_C$

And $x_A + x_B + x_C = \frac{3}{2}L\alpha\Delta T$

$2x_B + x_B + 2x_B = 5x_B = \frac{3}{2}L\alpha\Delta T \quad \Rightarrow \quad x_B = \frac{3}{10}L\alpha\Delta T$

Energy stored $E = \frac{1}{2}kx_A^2 + \frac{1}{2}(2k)x_B^2 + \frac{1}{2}kx_C^2$

$$=\frac{1}{2}k\left(2x_B\right)^2+kx_B^2+\frac{1}{2}k\left(2x_B\right)^2=2k\left(x_B^2\right)+kx_B^2+2k\left(x_B^2\right)=5kx_B^2$$

$$E=5k\left(\frac{9}{100}L^2\alpha^2\Delta T^2\right)=\frac{9}{20}k\alpha^2L^2\Delta T^2=.45k\alpha^2L^2\Delta T^2$$

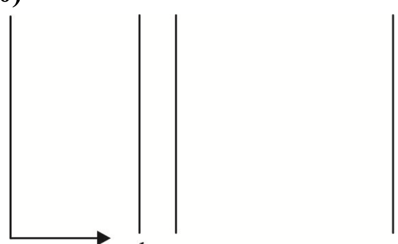
15.(3.12)

$$(180)^2+(255-15t)^2=(300t)^2$$

$\therefore \quad t=1s$

Also $f'=\left(\frac{300+15\cos 37}{300-10\cos 53^\circ}\right)f_0=\frac{312}{294}\times 2.94=3.12\,Hz$

16.(2.00)



$$dC_1=\frac{A\left(\in_0+\frac{\beta\in_0 x}{d}\right)}{dx}$$

$$\frac{1}{C_1}=\int\frac{1}{dC_1}=\int_0^{d/2}\frac{dx}{A\in_0\left(1+\frac{\beta x}{d}\right)}=\frac{d}{A\in_0\beta}\ln\left[1+\frac{\beta}{2}\right]$$

$$\frac{1}{C_2}=\int\frac{1}{dC_2}=\int_{d/2}^{d}\frac{dx}{A\in_0\left(1+\frac{\beta(d-x)}{d}\right)}=\frac{d}{A\in_0\beta}\ln\left[1+\frac{\beta}{2}\right]$$

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{2d}{A\in_0\beta}\ln\left[1+\frac{\beta}{2}\right]$$

It is given $C_{eq}=\frac{A\in_0\beta}{2d\ln\left(1+\frac{\beta}{2}\right)}=\frac{1}{2\ln 2}\times\frac{A\in_0\beta}{d}$

17.(8) Using conservation of energy. $420=\left(m\times 10^{-3}\right)(2100)(5)+\left(1\times 10^{-3}\right)\left(3.36\times 10^5\right)$

Solving this equation, we get $m=8g$ $\therefore$ The correct answer is 8.

18.(2) $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad\Rightarrow\quad \frac{1}{v}+\frac{1}{-30}=\frac{1}{-20} \quad\Rightarrow\quad \frac{1}{v}=\frac{1}{30}-\frac{1}{20}=-\frac{1}{60}$

$v=-60$ cm $\quad\Rightarrow\quad |Y|=10$ cm $\quad\Rightarrow\quad N=2$

CHEMISTRY

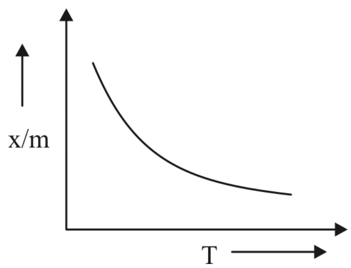
1.(D) Borax bead contains $NaBO_2 + B_2O_3$

2.(B) **Bauxite** $Al_2O_3.2H_2O$

Siderite $FeCO_3$

Argentite Ag_2S

3.(C)

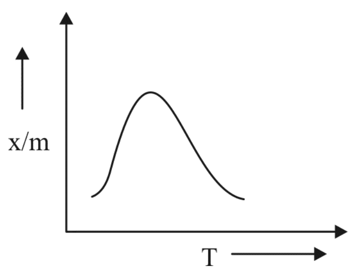


Physical adsorption

$E_a = 0$; $\Delta H = -ve$

So, according to Le chatlier principle

$\frac{x}{m} \propto \frac{1}{\text{temperature}}$



Chemical adsorption

Extent of adsorption first increases up to a certain temperature and then desorpton is favoured at high temperature. Initial increase in the extent of chemisorption with temperature is because some E_a is required to cross energy barrier.

4.(D) $NaHCO_3$ is weak base, which does not show acidic nature of phenol, but show acidic nature of $-COOH$ group.

5.(ABC)

The compound z is borax $(Na_2B_4O_7)$. It is prepared by boiling finely powdered colemanite mineral $(Ca_2B_6O_{11})$ with sodium carbonate solution.

$Ca_2B_6O_{11} + 2Na_2CO_3 \rightarrow 2CaCO_3 + Na_2B_4O_7 + 2NaBO_2$

A. The structure of anion of crystalline z has two boron atoms sp^3 hybridized and other two boron atoms sp^2 hybridized.

B. $2NaBO_2 + 2H_2O_2 + 6H_2O \longrightarrow Na_2\left[B_2(O_2)_2(OH)_4\right].6H_2O$

C. $Na_2B_4O_7 + H_2SO_4 \longrightarrow Na_2SO_4 + H_2B_4O_7$

$H_2B_4O_7 + 5H_2O \longrightarrow 4H_3BO_3$

$H_3BO_3 + 4HF \longrightarrow H[BF_4] + 3H_2O$

D. $B_2O_3 + CoO \longrightarrow \underset{\text{cobalt metaborate}}{Co(BO_2)_2}$

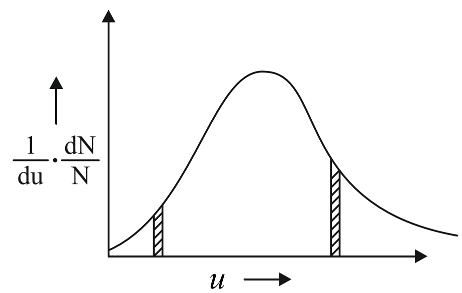
(blue bead)

6.(BCD)

Colour of anhydrous $CuSO_4$ is white.

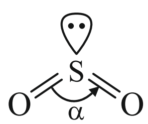
7.(BD) Diamond gives more heat on complete combustion.

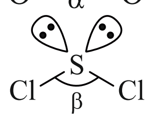
8.(ABC)



Fraction of molecule between speed u and $u + du$.

9.(ABC)

$AX_2 : SO_2$  $\rightarrow$ Hyb. of S: sp^2; shape : Bent

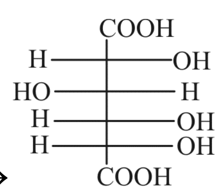
$AY_2 : SCl_2 \rightarrow$  $\rightarrow$ Hyb. of S: sp^3; shape : Bent $\rightarrow \alpha > \beta$

$\rightarrow$ Both are planar

10.(ABC)

Receptors show selectivity for one chemical messenger over the other because their binding sites have different shape, structure and amino acid composition. These receptors receive chemical messengers at binding sites.

11. (AB)

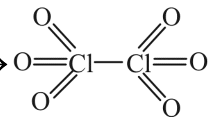
(C) Glucose $+HNO_3 \rightarrow$ 

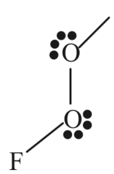
(D) Mutarotation is a general property of cyclic (chiral) hemiacetals.

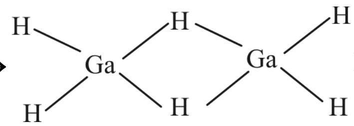
12.(BD) Aspirin inhibits the synthesis of prostaglandins.

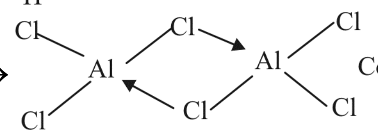
Antibiotics in low concentration inhibits the growth of microorganism by intervening in their metabolic processes.

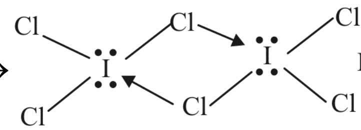
Antibiotics(chloramphenicol) are used as drug because of its low toxicity for humans and animals.

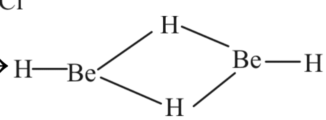
13.(3) $2ClO_3 \xrightarrow[\text{dimerisation}]{\text{cool}}$  Hypervalent

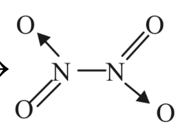
$2OF \xrightarrow[\text{dimerization}]{\text{cool}}$  Completion of octet

$2GaH_3 \xrightarrow[\text{dimerization}]{\text{cool}}$  Hypovalent

$2AlCl_3 \xrightarrow[\text{dimerization}]{\text{cool}}$  Completion of octet

$2ICl_3 \xrightarrow[\text{dimerization}]{\text{cool}}$  Hypervalent

$2BeH_2 \xrightarrow[\text{dimerization}]{\text{cool}}$  Hypovalent

$2NO_2 \xrightarrow[\text{dimerization}]{\text{cool}}$  Completion of octet value of $\frac{yz}{x} = \frac{3\times 2}{2} = 3$

14.(7) Incorrect reactions are

(ii) $CuSO_4 + PH_3 \longrightarrow \underset{(\text{Black ppt})}{Cu_3P_2}$

(viii) $SbF_5 + XeF_4 \rightarrow [XeF_3]^+[SbF_6]^-$

15.(2) Let's assume simultaneous solubility of $SrCO_3$ as xM while SrF_2 as yM

$$SrCO_3(s) \rightleftharpoons Sr^{2+}(aq) + CO_3^{2-}(aq)$$

At eq. $\quad (x+y)M \quad xM \quad K_{sp_1} = (x+y)x$

$$SrF_2(s) \rightleftharpoons Sr^{2+}(aq) + 2F^-(aq);\ 2.5\times 10^{-10} = (x+y)x$$

At eq. $\quad (y+x)M \quad 2y\,M; \quad K_{sp_2} = (x+y)(2y)^2$

$10^{-10} = 4y^2(x+y)$

$2.5 = \frac{x}{4y^2} \quad \Rightarrow \quad 10y^2 = x = 10^{-3} \quad \Rightarrow \quad y^2 = 10^{-4}$

$y = 10^{-2}M \quad \therefore \quad [F^-] = 2y = 2\times 10^{-2}M$

16.(81.10)

$$\Delta T_b = k_b \times m;\ m = \text{molality} = \frac{n_{solute}}{W_{solvent}(\text{in kg})}$$

$$\Delta T_b = 2.6 \times \frac{15.4 \times 1000}{154 \times 260};\qquad \Delta T_b = 1$$

The boiling point of the resulting solution is $80.1 + 1 = 81.1°C$

17.(18.40)

Initial m mol of cyclobutene = a

Let after 20 min, x m mol cyclobutene isomerized.

m mol of cyclobutene left $= a - x$ and m mol of diene formed $= x$

Number of millimoles of Br_2 required $=$ Number of millimoles of π bond in substance

m mol of Br_2 required after 20 min $= a - x + 2x = a + x = 16$

$\Rightarrow$ $2a = 20$ (only 1, 3-butadiene present)

$a = 10$ and $x = 6$

$\therefore$ $a = 10,\ 20k = \ln\frac{10}{4}$ ...... (i)

If y m mol of cyclobutene isomerized after 40 min.

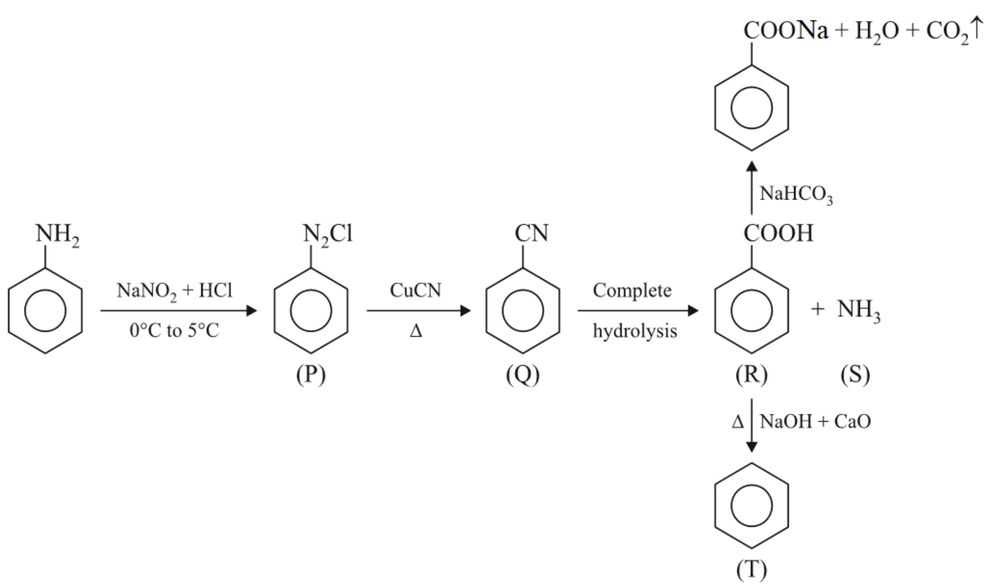
$$40k = \ln\frac{10}{10 - y} \qquad \text{...... (ii)}$$

From equation (i) and (ii) $y = 8.4$

$\Rightarrow$ m mol of Br_2 required $= 10 + 8.4 = 18.4$

$\Rightarrow$ Volume of bromine solution required $= 18.40\,\text{mL}$

18.(78)



MATHEMATICS

1.(A) **Given :** $a|z_1| = b|z_2| \Rightarrow \frac{az_1}{bz_2} = \frac{b\overline{z}_2}{a\overline{z}_1}$

Now $\frac{az_1}{bz_2} + \frac{bz_2}{az_1} = \frac{b\overline{z}_2}{a\overline{z}_1} + \frac{bz_2}{az_1} = \frac{b}{a}\left(\frac{\overline{z}_2}{\overline{z}_1} + \frac{z_2}{z_1}\right)$ = Real number in [–1, 1]

2.(B) $M = (A_1) + 3(A_2)^3 + 5(A_3)^5 \ldots\ldots + 15(A_8)^{15}$

$M^T = \left(A_1^T\right) + 3\left(A_2^T\right)^3 + 5\left(A_3^T\right)^5 \ldots\ldots + 15\left(A_8^T\right)^{15}$

$M^T = -M$

Skew symmetric

3.(C) Let the midpoint of a chord be $M(\alpha, \beta)$ then the equation of chord i.e. $T = S_1$

$\Rightarrow \quad x\alpha + y\beta - \gamma^2 = \alpha^2 + \beta^2 - \gamma^2$ passes through (p, q)

$\Rightarrow \quad \alpha^2 + \beta^2 - p\alpha - q\beta = 0$

Locus is $x^2 + y^2 - px - qy = 0$

$\Rightarrow \quad h = 0, 2g = -p, 2f = -q, c = 0$

4.(D) $A_n = \int_0^n \left(([x]) + \sqrt{\{x\}} - \left([x] + \{x\}^2\right)\right)dx = \int_0^n \left(\sqrt{\{x\}} - \{x\}^2\right)dx = n\int_0^1 \left(\sqrt{x} - x^2\right)dx = \frac{n}{3}$

5.(BD) For $P(x) = 0$

$Q(Q(x)) - x = 0$

$(Q(x)) - x$ must be a factor

Let $t = Q(x) - x \Rightarrow Q(x) = x + t$

Now $Q(Q(x)) - x = 0$

$\Rightarrow \quad (x+t)^2 - 2(x+t) + 3 - x = 0 \quad \Rightarrow \quad t^2 + 2xt + x^2 - 2x - 2t + 3 - x = 0$

$\Rightarrow \quad t^2 + 2xt - 2t + t = 0 \quad \Rightarrow \quad t[t + 2x - 1] = 0 \quad \Rightarrow \quad (x^2 - 3x + 3)(x^2 - x + 2) = 0$

$\Rightarrow$ all four non-real roots and $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{\beta + \alpha}{\alpha\beta} + \frac{\delta + \gamma}{\gamma\delta} = \frac{3}{3} + \frac{1}{2} = \frac{3}{2}$

6.(BCD)

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -\lambda & 3 & 2 \\ 0 & -2-\lambda & 0 \\ 1 & 2 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda^3 + \lambda^2 - 4\lambda - 4 = 0 \quad \Rightarrow \quad A^3 + A^2 - 4A - 4I = 0$

$\Rightarrow \quad 4A^{-1} = A^2 + A - 4I \quad \Rightarrow \quad a = 1, b = 1, c = -4$

7. (BCD)

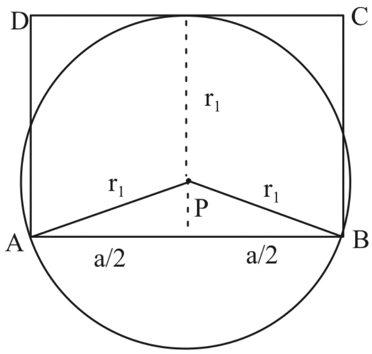
$$E_1 = (2,2),(3,3),(5,5),(2,3),(3,2),(2,5),(5,2),(3,5),(5,3)$$

$$E_2 = (1,1),(2,2),(3,3),(4,4),(5,5),(6,6)$$

$$E_3 = (1,4),(4,1),(2,3),(3,2)$$

$$p(E_1) = \frac{9}{36}, p(E_2) = \frac{6}{36}, p(E_3) = \frac{4}{36}$$

8.(BD)

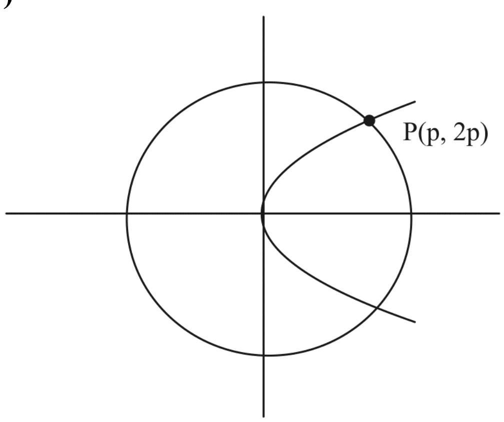


$r_1 + p = b$ and $r_1^2 = p^2 + \frac{a^2}{4}$

$$\Rightarrow \quad r_1^2 = (b - r_1)^2 + \frac{a^2}{4}$$

$r_1 = \frac{4b^2 + a^2}{8b}$ similarly $r_2 = \frac{4a^2 + b^2}{8a}$

9.(ABD)



$P(p, 2p)$ on end point of latus rectum of parabola $y^2 = 4px$ lies at

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{p^2}{a^2} + \frac{4p^2}{b^2} = 1 \ldots\ldots(i)$$

$\because$ tangent at P to both the curves are perpendicular to each other

$$\Rightarrow \quad \frac{2p}{2p}\cdot\left(\frac{-b^2}{a^2}\cdot\frac{p}{2p}\right)=-1$$

$$\Rightarrow \quad b^2=2a^2 \quad \Rightarrow \quad e=\frac{1}{\sqrt{2}} \qquad \ldots\ldots \text{(ii)}$$

And $\left(e,\frac{b}{a}\right)=\left(\frac{1}{\sqrt{2}},\sqrt{2}\right)$ lies at E

$$\Rightarrow \quad \frac{1}{10}+\frac{2}{12}=k^2 \quad \Rightarrow \quad k^2=\frac{8}{30}=\frac{4}{15}$$

10.(CD)

$f''(x)>0 \Rightarrow f'(x)$ is an increasing function i..e., $f'(x_2)>f'(x_1) \Rightarrow x_2>x_1$

Now $g'(x)=3.f'\left(\frac{x^2}{3}\right).\frac{2x}{3}-2xf'\left(12-x^2\right)=2x\left(f'\left(\frac{x^2}{3}\right)-f'\left(12-x^2\right)\right)$

If $\frac{x^2}{3}>12-x^2 \Rightarrow x^2>9$

$g'(x)>0 \quad \Rightarrow \quad x>0$ and $x^2=9$ $or\, x<0$ $and\, x^2<9$

$\Rightarrow \quad g(x)$ is increasing in $(-3,0)\cup(3,\infty)$

$\quad g(x)$ is decreasing in $(-\infty,-3)\cup(0,3)$

11.(BD)

$f(x)+g'(x)=1$ and $f'(x)+g(x)=1$

$f'(x)+g'(x)+f(x)+g(x)=2$

$e^x\left(f'(x)+g'(x)\right)+e^x\left(f(x)+g(x)\right)=2e^x$

Integrating both sides $\Rightarrow e^x\left(f(x)+g(x)\right)=2e^x+c$

As $x=0,\ f(0)=2,\ g(x)=1 \Rightarrow c=1$

$$\Rightarrow \quad f(x)+g(x)=2+e^{-x} \qquad \ldots\ldots \text{(i)}$$

Again $f(x)+g'(x)=1$ and $g(x)+f'(x)=1$

$$\Rightarrow \quad f'(x)-g'(x)=f(x)-g(x)$$

$$\Rightarrow \quad f(x)-g(x)=e^x \qquad \ldots\ldots. \text{(ii)}$$

$$\Rightarrow \quad f(x)=\frac{1}{2}\left[e^x+e^{-x}+2\right],\ g(x)=\frac{1}{2}\left[2+e^{-x}-e^x\right]$$

12.(AB) $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

$2\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\vec{b}+\vec{c}$

$2(\vec{a}.\vec{c})\vec{b}-2\left(\vec{a}.\vec{b}\right)\vec{c}=\vec{b}+\vec{c} \quad \Rightarrow \quad \vec{a}.\vec{c}=\frac{1}{2}$ and $\vec{a}.\vec{b}=-\frac{1}{2}$

13.(3) $\left|a+b\omega+c\omega^2\right| = \left|a-\left(\frac{b}{2}+\frac{c}{2}\right)+\frac{i\sqrt{3}}{2}(b-c)\right| = \sqrt{\left(a-\frac{b+c}{2}\right)^2+\frac{3}{4}(b-c)^2}$

Least value of $(b-c)^2 = 4$. $\quad$ $[b,\ c \to \text{odd integer}]$

$\therefore \quad b,\ c \to$ consecutive odd

Let $\quad b = 1,\ c = 3;\quad \left|a+b\omega+c\omega^2\right|_{\min} = \sqrt{(a-2)^2+3}$

Therefore, we can take $a = 5$ for minimum value.

$\therefore \quad \left|a+b\omega+c\omega^2\right|_{\min} = \sqrt{3^2+3} = 2\sqrt{3}$

14.(9) a, a^2 and b are in G.P. $\Rightarrow\ b = a^3$

a, b and a^2 are in A.P. $\Rightarrow 2a^3 = a + a^2$

$a \neq 0, 2a^2 - a - 1 = 0 \Rightarrow a = 1, a = -\frac{1}{2}$

a is negative $\Rightarrow a = -\frac{1}{2}$; $\qquad$ sum of ∞ CP $= \frac{a}{1-r} = \frac{a}{1-a} = \frac{-1/2}{1-\left(-\frac{1}{2}\right)} = -\frac{1}{3}$

15.(5) Total probability of getting late $= \frac{1}{4}\times\frac{2}{3}+\frac{3}{4}\times\frac{1}{3} = \frac{5}{12}$

Required probability $= 1-\left(\frac{5}{12}\right)^2 = \frac{119}{144}$

16.(5) $\because \quad$ PACB is a cyclic quadrilateral

D $=$ circumcentre of quadrilateral PACB $=$ mid point of P and C $= (3,4)$

$\Rightarrow \quad OD = 5$

17.(3) $f(x)+f\left(1-\frac{1}{x}\right) = x$

Replace x with $\left(1-\frac{1}{x}\right)$

$\Rightarrow \quad f\left(1-\frac{1}{x}\right)+f\left(\frac{1}{1-x}\right) = 1-\frac{1}{x}$

Again $f\left(\frac{1}{1-x}\right)+f(x) = \frac{1}{1-x}$

$\Rightarrow \quad 2f(x) = x+\frac{1}{1-x}-\left(1-\frac{1}{x}\right)$

$$\int_0^1 x(1-x)f(x)dx = \frac{1}{2}\int_0^1 \left(1-x+2x^2-x^3\right)dx = \frac{1}{2}\left[1-\frac{1}{2}+\frac{2}{3}-\frac{1}{4}\right] = \frac{11}{24} = \frac{a}{3b} \Rightarrow a-b=3$$

18.(7) Normal vector to the plane is $= \begin{vmatrix} i & j & k \\ 2 & 0 & 1 \\ 1 & 2 & -2 \end{vmatrix} = -2\hat{i}+5\hat{j}+4\hat{k}$

Equation of plane is $2x-5y-4z=\lambda$

Which pass through $(2,3,-4) \Rightarrow \lambda = 5$

Equation of plane is $-2x+5y+4z+5=0$

$\Rightarrow \quad a+b+c=7$